

## COMPARATIVE EVALUATION OF LACTATION CURVE MODELS FOR FIRST LACTATION MONTHLY MILK YIELD IN CROSSBRED DAIRY CATTLE

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### ABSTRACT

The study of lactation curve facilitates prediction of total milk yield from partial yield. Relevant information about daily farm practices can be acquired by studying the shape of lactation curve. Dairy cattle with flatter lactation curve are more persistent milker than with a steeper lactation curve. Data pertaining to monthly milk yield (MMY) records upto 300 days of lactation of 529 crossbred cows spread over a period of 30 years (1990-2019) were collected from Instructional Dairy Farm of G.B. Pant University of Agriculture and Technology, Pantnagar (Uttarakhand). Data under present investigation were examined by using four statistical models viz. Parabolic Exponential Function (PEF), Inverse Polynomial Function (IPF), Gamma Type Function (GTF) and Mixed Log Function (MLF) to determine the most suitable non-linear model. The model with the highest degree of coefficient of determination ( $R^2$ ), lowest value of root mean square error (RMSE) and absolute mean deviation (AMD) were contemplated as the best fitted model. The predicted MMY upto 300 days were very close to the observed MMY upto 300 days of lactation. However, among all the four models described above, IPF showed the highest degree of accuracy with the maximum coefficient of determination ( $R^2 = 97.1\%$ ), minimum root mean square error (RMSE = 5.10 Kg) and absolute mean deviation (AMD = 3.42 Kg) suggesting IPF to be the best fitted function for explaining the first lactation monthly milk yield records in the crossbred cattle.

**Keywords:** Monthly Milk Yield (MMY), Parabolic Exponential Function (PEF), Inverse Polynomial Function (IPF), Gamma Type Function (GTF), Mixed Log Function (MLF)

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Lactation curve modelling provides valuable guidelines in designing managerial and breeding strategies for dairy cattle. Lactation curve is a graphical representation of milk yield with respect to defined time period. Lactation curve consists of three phases of lactation: Ascending phase which stipulates that cow is maintained on high plane of nutrition; persistency phase depicting an inherent ability of a dairy animal to sustain the milk production after attaining peak milk yield and declining phase which stipulates that cow is maintained on low plane of nutrition. Appropriate findings of lactation curves for first lactation period provide valuable information about persistency of milk yield, managerial and breeding strategy and can also be used to predict complete milk yield with minimum error from partial milk yield records. Different mathematical models have been designed to analyse lactation curve but the first mathematical model was designed by Brody *et al.* (1923) which was a decreasing exponential function to fit declining phase of lactation in dairy cattle. Fitting of lactation curve models with the milk yields are necessary to get an idea about the milk pattern. Under the present investigation, four non-linear models have been used to predict the shape of lactation curve.

### MATERIALS AND METHODS

**Location:** The study was conducted on crossbred cattle maintained at Instructional Dairy Farm of G.B. Pant

University of Agriculture and Technology, Pantnagar, India. Farm is located at 28° 52' N to 28° 25' N latitude and 78° 58' to 79° 42' E longitude. Subtropical climate is prevalent in the region. Maximum temperature during summer has been recorded as 43.5 °C and minimum temperature as -0.8 °C in the winter with minimum and maximum rainfall of 0 mm and 218.6 mm, respectively.

**Herd management practices:** Animals were maintained under loose housing system. NRC feeding standards was adopted for feeding animals at the farm. Ration comprised of dry and green in the ratio of 2:1 supplemented with balanced proportion of concentrates, vitamins, minerals, probiotics and protein as a prophylactic measure to prevent occurrence of deficiency and metabolic diseases. Crossbreeding and selective breeding were the breeding methods adopted at the farm. At the farm crossbreeding was practiced since 1968 using semen of exotic breeds like HF, Jersey and Red Dane with indigenous breeds Sahiwal, Red Sindhi, Rathi, etc. and records have been maintained on the herd registers. Milking was done twice a day i.e., morning and evening and daily record of milk obtained was maintained in milk record registers by farm personnel.

**Data sources:** Data on 529 daughters sired by 79 sires were collected from Instructional dairy farm of G.B. Pant University of Agriculture and Technology, Pantnagar (Uttarakhand). The first lactation monthly milk yield for 300 days of lactation was taken into consideration under

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the present investigation. Milk yield lesser than 300 days was not considered while collecting data for the present study.

**Statistical analysis:** The data for first lactation traits were analysed using Mixed Model Least Squares and Maximum Likelihood Programme of Harvey (1990). The lactation curve parameters were calculated using statistical analysis software version 9.3 (SAS institute Inc. 2011) and multiple regression analysis was used to formulate the equations to be fitted into the mathematical models as presented in Table 3. The shape of lactation curve was predicted using four statistical models *viz.* parabolic exponential function, inverse polynomial function, gamma type function and mixed log function summarised as below:

1. Parabolic exponential function (Sikka, 1950)

$$Y_t = A \cdot e^{bt+ct^2}$$

After taking natural logarithm and linear regression model the equation becomes:  $\log_e Y_t = \log_e A + bt + ct^2$ ; or  $z = A + bt + ct^2$ ; or  $z = A + b_1 t_1 + b_2 t_2$  where,  $z = \log_e Y_t$ ;  $A = \log_e A$ ;  $b_1 = b$ ;  $b_2 = c$ ;  $t_1 = t$  and  $t_2 = t^2$

2. Inverse polynomial function (Nelder, 1966)

$$Y_t = t(b_0 + b_1 t + b_2 t^2)^{-1}$$

After dividing both the sides by  $t$  and taking reciprocal the above equation can be written as:

$$t/Y_t = b_0 + b_1 t + b_2 t^2$$

3. Gamma type Function (Wood, 1967)

$$Y_t = A t^b e^{-ct}$$

By taking natural logarithm on both sides the equation becomes:  $\log_e Y_t = \log_e A + b \log_e t - ct$ ; or  $\log_e Y_t = \log_e A + b \log_e t - ct$ ; or  $z = A + b_1 X_1 + b_2 X_2$

where,  $z = \log_e Y_t$ ;  $A' = \log_e A$ ;  $b' = b$ ;  $X_1 = \log_e t$ ;  $b_2 = -c$  and  $X_2 = t$

4. Mixed log function (Guo and Swalve, 1995)

$$Y_t = A + b\sqrt{t} + c \log t$$

Where,  $Y_t$  = average daily yield in the  $t$ th month of lactation;  $A$ =approximate initial milk yield just after parturition;  $e$ =exponential constant (2.71828);  $b$ = inclining slope parameter up to peak yield;  $b_0$  = observed value at the time of parturition;  $b_1$  = rising extremes of the curve;  $b_2$  = declining extremes of the curve;  $c$ =declining slope parameter (persistence measure);  $t$  = length of time since parturition. To judge the efficacy of above four statistical models, coefficient of determination ( $R^2$ ), Root mean square error (RMSE) and Absolute mean deviation (AMD) were used as goodness of fit tests.

## RESULTS AND DISCUSSION

The observed and predicted average monthly milk yields along with their errors for first lactation milk yield in crossbred cattle are presented in Table 1. The observed monthly milk yield at first month after calving was 255.22 kg. Increased milk yield of 316.48 kg was observed at second month after calving thereafter a gradual decline in milk yield was observed upto ten months after calving with tenth month showing the lowest monthly milk yield of 241.49 kg. Lactation curves for parabolic exponential function, inverse polynomial function, gamma type function, mixed log function and all prediction functions are presented in figure 5. Coefficient of determination ( $R^2$ ), root mean square error (RMSE) and absolute mean deviation (AMD) were the goodness of fit tests considered to select the best lactation curve fitting model (Table 3).  $R^2$  values were sufficiently high for inverse polynomial function (97.1%) followed by mixed log function (92.6%), gamma type function (89.2%) and parabolic exponential function (69.1%). RMSE values were highest for the parabolic exponential function (17.01 Kg) followed by gamma type function (10.07 Kg), mixed log function (8.03 Kg) and inverse polynomial (5.10 Kg). AMD values were highest for parabolic exponential function (11.87 Kg) followed by gamma type function (7.20 Kg), mixed log function (5.46 Kg) while lowest for the inverse polynomial (3.42 Kg) function. The models that gave the highest degree of  $R^2$  have been accepted as the best lactation curve fitting models. Therefore, IPF with highest value of  $R^2$  could adequately fit the monthly milk yield data for 300 days of lactation while MLF, GTF and PEF gave reliable information about lactation curve pattern in crossbred cattle.

The inverse polynomial function was the best fitted model for explaining the lactation curve and this was also opined by Singh (1973) for explaining individual lactation curve, Roy (1983) in Sahiwal×Jersey cattle using individual milk records, Gahlot *et al.* (1989) in Rathi×Red Dane crosses, Roy and Katpal (1993) in Jersey cattle, Kumar (1994) in CB cattle, Kumar *et al.* (1996) in Jersey×Sahiwal half-bred cows, Kumar *et al.* (1997) in HF×S half-bred cows, Singh *et al.* (1998) in Jersey×Sahiwal F1 cows, Gandhi and Dongre (2013) in Sahiwal cattle, Sohal (2016) in Rathi cattle and Arya *et al.* (2020) in various crossbred dairy cows. However, Singh (1973) reported gamma type function as the best fitted model for explaining first lactation records in dairy cattle. Kumar (1994) concluded gamma type and parabolic exponential function followed by inverse polynomial and exponential as the best fitting model to estimate average monthly/weekly milk records in Jersey×Sahiwal cattle. Singh *et al.* (1996) reported gamma

**Table 1. Predicted monthly milk yields and errors of different models**

Part milk yield (days)	Observed milk yield (Kg)	Parabolic Function		Inverse polynomial function		Gamma type function		Mixed log function	
		Predicted milk yield (Kg)	Error	Predicted milk yield (Kg)	Error	Predicted milk yield (Kg)	Error	Predicted milk yield (Kg)	Error
30	255.22	282.97	-27.75	259.61	-4.39	266.5	-11.28	262.82	-7.6
60	316.48	292.67	23.81	307.54	8.94	299.48	17	302.34	14.14
90	315.68	298.52	17.16	314.26	1.42	308.81	6.87	310.24	5.44
120	306.02	300.29	5.73	308.16	-2.14	307.4	-1.38	307.3	-1.28
150	291.64	297.89	-6.25	297.51	-5.87	300.15	-8.51	299.33	-7.69
180	281.51	291.43	-9.92	285.4	-3.89	289.51	-8	288.68	-7.17
210	271.2	281.16	-9.96	273.1	-1.9	276.91	-5.71	276.48	-5.28
240	262.15	267.52	-5.37	261.17	0.98	263.23	-1.08	263.37	-1.22
270	254.56	251.02	3.54	249.84	4.72	249.05	5.51	249.7	4.86
300	241.49	232.28	9.21	239.2	2.29	234.76	6.73	235.7	5.79

**Table 2. Lactation curves parameters of different models for prediction of monthly milk yields**

S.No.	Models	Parameters					
		A	b	c	b <sub>0</sub>	b <sub>1</sub>	b <sub>2</sub>
1.	Parabolic exponential	269.809	0.055	0.007	-	-	-
2.	Inverse polynomial	-	-	-	0.002	0.002	0.000
3.	Gamma type	291.774	0.299	0.091	-	-	-
4.	Mixed log	464.296	-201.478	177.422	-	-	-

A = approximate initial milk yield just after parturition; b = inclining slope parameter up to peak yield; c = declining slope parameter (persistency measure); b<sub>0</sub> = observed value at the time of parturition; b<sub>1</sub> = rising extremes of the curve; b<sub>2</sub> = declining extremes of the curve.

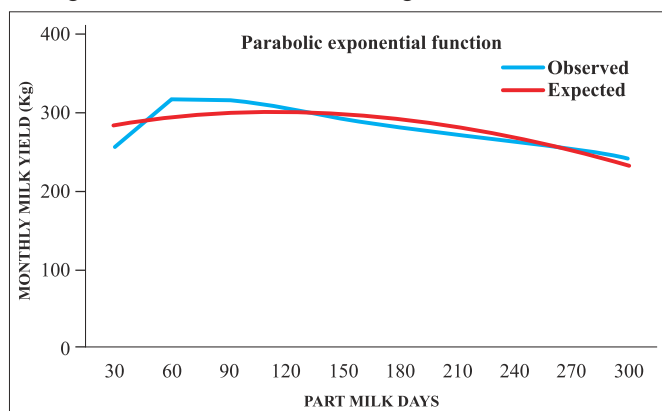


Fig.1. Predicted monthly milk yields by parabolic exponential function

type function as the best fitted model followed by inverse polynomial and parabolic exponential in crossbred cattle. Singh *et al.* (1997) opined gamma stochastic and gamma type function to be the best fitted curve followed by parabolic exponential, inverse polynomial and exponential function in Jersey×Sahiwal crossbred cows and Daltro *et al.* (2021) found Wood’s model to be best curve fitting model for explaining monthly milk yield in Holstein×Gir

**Table 3. Different models with their prediction equations and goodness of fit tests**

S.No.	Models	Prediction equations	R <sup>2</sup> (%)	RMSE (Kg)	AMD (Kg)
1	Parabolic exponential	$Y_t = 269.80 \cdot e^{0.055t + 0.007t^2}$	69.1	17.01	11.87
2	Inverse polynomial	$Y_t = t(0.002 + 0.002t)^{-1}$	97.1	5.10	3.42
3	Gamma type	$Y_t = 291.77t^{0.29}e^{-0.09t}$	89.2	10.07	7.20
4	Mixed log	$Y_t = 464.29 + [-201.47\sqrt{t} + 177.42 \log t]$	92.6	8.03	5.46

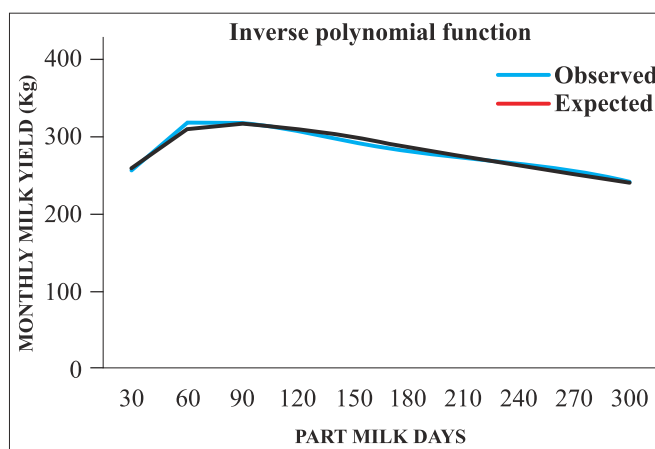


Fig.2. Predicted monthly milk yields by inverse polynomial function cattle.

**CONCLUSION**

It was concluded that the estimated lactation curve parameters can be effectively used for the selection and improvement of dairy cattle with high persistency and total lactation milk yield. The IPF followed by MLF, GTF and PEF depicted best fit to the data of monthly milk yield from 30 to 300 days of lactation for accurate prediction and characterisation of lactation curve patterns for first lactation milk yield. However, IPF described the highest

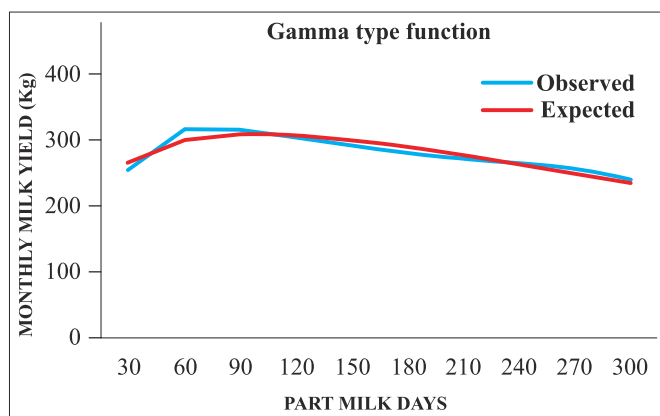


Fig. 3. Predicted monthly milk yields by gamma type function

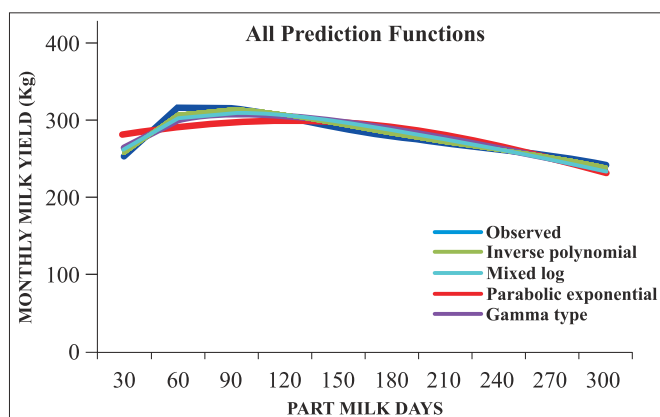


Fig. 5. Comparative evaluation of predicted monthly milk yields by all four functions

degree of  $R^2$ , minimum value of RMSE and minimum value of AMD among all the four models. Therefore, IPF is the best fitted model for the data under the present study and can be used to design managerial and breeding strategies at the farm level.

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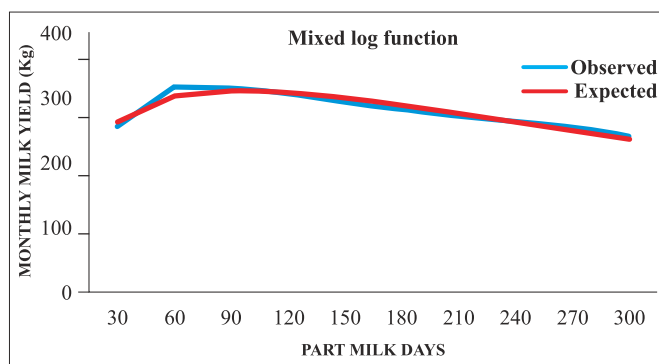


Fig. 4. Predicted monthly milk yields by mixed log function

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